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unpublished work by Goodwin and Wilson of the Massachusetts Institute of Technology. In figure 2 the solid line shows the experimentally determined variation with the pressure of the overvoltage of nickel. Similar curves were found for mercury and lead. The dotted line shows the variation as calculated by Equation 2, using the overvoltage at one atmosphere as a basis for computing the values for the other pressures. The difference between these two curves may be explained by an increase of stirring at the lower pressures, since many more bubbles are produced per mol of gas.

It appears quite probable, then, that the factor that determines the overvoltage of an electrode at any one pressure is the size of the gaseous nuclei that can cling to it. A number of observers have called attention to the fact that electrodes with low overvoltages are those that have large adsorptive powers. This adsorptive power is undoubtedly related to the attraction of an electrode for a gaseous nucleus.

AN APPROXIMATE LAW OF ENERGY DISTRIBUTION IN THE GENERAL X-RAY SPECTRUM

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In the spectra of X-rays as ordinarily determined there are factors of absorption in the anticathode, the glass of the tube, the reflecting crystal and the ionized gas, and of efficiency of reflection that are all functions of the frequency. Fortunately, except at the discontinuities of any of these absorptions, the unknown factors vary continuously with frequency, so that the measured intensities in the spectrum represent the energy distribution qualitatively, but by no means quantitatively. The problem of the present paper is to combine other available data in such a way as to find an approximate law of energy distribution, not involving unknown absorption factors, and avoiding also any *a priori* assumptions about the emitting mechanism. The data are incomplete and this work is merely a first approximation.

For data we have (a) some graphs of intensity against potential at constant frequency (where the unknown factors are all constant in each graph), and (b) the total energy measurements by Beatty,[1] who made the absorption negligible by using a thin window and no crystal. Some of the intensity-potential graphs were obtained in the course of experiments for another purpose with a rhodium target by the author,[2] and with platinum by the author and Dr. H. Clark,[3] and others were obtained by taking points at the same wave length from intensity-wave length graphs drawn for tungsten by A. W. Hull,[4] Hull and Rice[5] and Ulrey,[6] and for molybdenum by Hull.

In the experiments on rhodium and platinum, the spectrometer was kept at a fixed wave length and the potential was changed between readings.

This gave directly a series of intensity-potential graphs that are represented with fairly good accuracy by the law

$$I(V,\nu) = k(\nu)\left\{(V - H\nu) + H\nu p(\nu)\left(1 - e^{-q(\nu)\left(\frac{V}{H\nu}-1\right)}\right)\right\},$$

where $I\ (V,\nu)d\nu$ is the intensity in the frequency range $d\nu$ per electron striking the target from potential V, and H is the ratio of Planck's h to the charge of the electron, and $k(\nu)$, $p(\nu)$ and $q(\nu)$ are functions of ν. Since p and q are pure numbers they are independent of the arbitrary intensity unit, and can be determined wherever the data are available, though not very accurately because the term containing them is rather small. In the few data available for rhodium, p is of the order of 0.06 to 0.08 and q about 12 to 16, making pq about 1. The work on tungsten and molybdenum gives only a few points on each intensity potential graph, and because of the smallness of the exponential term and its disappearance at potentials large enough for really accurate intensity measurements, it is impossible to get an accurate test of this law except with more points than one can get from these graphs. But the data obtainable show that the relation between I and V is not far from linear, and the only definite curvature seems to be something of the type indicated by the above equation. In platinum, we have data scattered over the range from 1.33 to 0.43 Å, but most of them rather rough. But to an accuracy of 20 or 30%, they seem to indicate constant values for both p and q, with $p = 1/5$ and $q = 13$, so that $pq = 5/2$. Fortunately the smallness of the p and q terms makes their influence on the determination of k also small, although the existence of the terms themselves may be of considerable theoretical importance. For the present, therefore, we shall include these terms in the calculation, but neglect any changes of p and q with ν.

An important point to be deduced from the graphs of 'intensity' against ν is the fact that they are smooth and regular, so that k must have no discontinuities or sharp curvature in its graph against ν. As a trial value we shall therefore assume first that $k(\nu) = k\nu^n$, with k and n both constant. The total energy is then

$$E = \int_0^{\frac{V}{H}} I\,(V,\nu)d\nu = k\left\{\frac{1}{n+1} - \frac{1-p}{n+2} - p\int_0^1 x^{n+1}\cdot e^{-q\left(\frac{1}{x}-1\right)}dx\right\}\frac{V^{n+2}}{H^{n+1}},$$

If $k(\nu)$ is not expressible in this way, but as a power series in ν, then this equation gives a power series for E in terms of V, and if the coefficients for such a series are found experimentally those of the series for k can be computed from them. If E depends on a single power of V, then k must depend on a single power of ν.

Now, Beatty's work indicates that $E = \text{constant} \times V^2$. Hence, we infer that $n = 0$ and $k(\nu) = k = \text{constant}$, and

$$E = k\left\{\frac{1}{2} + \frac{p}{2} - p\int_0^1 xe^{-q\left(\frac{1}{x}-1\right)}\,dx\right\}\frac{V^2}{H},$$

or approximately

$$E = k\left\{\frac{1}{2} + \frac{p}{2} - \frac{p}{q}\right\}\frac{V^2}{H}.$$

In a previous paper by the author[7] it was shown that, neglecting absorption in the target, the intensity per electron and per atom and per unit interval of frequency from an extremely thin target would be

$$i\,(V, \nu) = \frac{bD_V I(V,\nu)}{2NV},$$

where b is the coefficient of the Thomson-Whiddington law in the form $V_0^2 - V_x^2 = bx$, and N is the number of atoms struck per unit length of the electrons' path. This equation is of course subject to some error due to absorption in the target, and as one may readily prove, could be corrected by adding to i a term

$$\frac{a\,\mu}{N}\,I\,(V, \nu),$$

where a is the ratio of the distance travelled in the target by the emerging X-rays to the distance travelled by the cathode ray before it emits X-rays, and μ is the linear absorption coefficient. This correction is important only at low frequencies. Neglecting it, we have

$$i\,(V, \nu) = \frac{bk}{2NV}\left\{1 + pqe^{-q\left(\frac{V}{H\nu}-1\right)}\right\},$$

with k, p and q all constant. It must be remembered that the data are very incomplete and this result is unreliable and is presented only for lack of anything better. But the true law must have something of the same general characteristics as this, and certain conclusions about the emitting mechanism can be drawn from that fact.

First, let us assume that some form of quantum law governs the radiation of frequencies different from V/H as well as at that one. We are then practically though not rigorously led to one of two alternatives. If the quantum law merely regulates the frequency being emitted at any instant in terms of the energy still available at that instant for radiation, so that the radiation by one electron is not monochromatic, then every electron radiating at all must radiate some energy at the highest possible frequency, V/H, and presumably give up all its energy to radiation. But this would make $i(V, \nu)$ independent of ν, and is therefore improbable. The more probable alternative is that the radiation by one electron is monochromatic, and the quantum law gives its frequency in terms of the total energy radiated. In this case only a few of the electrons radiating will do so at frequencies very near the limit, V/H, though more will radiate there than in an equal frequency interval at a frequency not too far below that one. Adopting this conclusion, the chance that an electron radiates a fraction of its energy within a range ϵ to $\epsilon + d\epsilon$ is $f(V, \epsilon)d\epsilon$, where

$$f(V, \epsilon) = \frac{bk}{2Nh}\,\frac{1 + pqe^{-q\left(\frac{1}{\epsilon}-1\right)}}{V\epsilon}.$$

Let R be the radius of the atom and r the distance from the particle causing radiation to the path the electron would have taken if not deviated. Then

$$\frac{d(r^2)}{R^2} = -f(V, \epsilon)\,d\epsilon, \text{ with } \epsilon = 1 \text{ at } r = 0,$$

or

$$\frac{r^2}{R^2} = \int_{\epsilon}^{1} f(V, \epsilon)\,d\epsilon.$$

If we assumed the frequency to be determined by the energy transferred in a collision of two equal repelling particles, one of which was initially at rest, a definite value of ν could be predicted for any assumed values of r and V, and the forms of $f(V, \epsilon)$ and $i(V, \nu)$ could be found. Several such assumptions have been tried, but none give the forms of f and i required above. A detailed discussion would be out of place with such rough preliminary data, and any such assumption would be improbable, but it seems likely that more exact data of this type ought to throw valuable light on the mechanism of radiation. It is hoped that with apparatus now being constructed it will be possible to get such data.

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[6] Ulrey, C. T., *Physic. Rev., Ithaca,* **11**, 1918, (401–10).

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